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# The minimum volume fraction of SiC reinforcement required for strength improvement of an Al–Cu based composite

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Metal matrix composites (MMCs) represent a unique class of materials with an ability to blend the properties of the ceramics with those of metals/alloys. The incorporation of hard and brittle ceramics phase is widely carried out in order to improve on the strength and wear properties of metallic matrices. In related studies carried out on spray-processed materials, it was observed that the incorporation of the ceramic particulates need not always increase the strength of metallic matrix unless they exceed a certain critical volume fraction. In order to explain these rather unusual results, a theoretical model was formulated to determine the critical volume fraction that may be required in order to realize the improvement in the ultimate tensile strength of the metallic matrices. The model proposed here is based on the equivalent inclusion method as related to the micromechanics of composites. The results derived from the model are interpreted with a view to establishing the link between the theoretical results obtained with the experimental findings.

## 1. Introduction

Metal matrix composites (MMCs), due to their unique ability to combine the properties of a ductile metallic matrix with that of hard and brittle ceramic reinforcement, have been actively sought to cater for a spectrum of applications [1]. One such application area that is presently under intensive investigation is to replace the conventional materials with metal matrix composites in the structural applications primarily related to automobiles and aerospace sectors [2]. In order to qualify for such applications, it is imperative that metal matrix composites should exhibit superior strength without significant loss in ductility over their monolithic counterpart. Various strengthening models have been proposed to explain the strengthening mechanism and predict the strength of MMCs [3–7]. One of the simplest ways to estimate the strength of a composite material is to use the rule of mixtures. This rule implies that the strength of the composite material will always be higher than that of the matrix material when the volume fraction of the ceramic reinforcement is greater than zero. However, it has been shown by various investigators that the addition of hard and brittle ceramic phase in the metallic matrix may not always increase the strength [8–11]. Ibrahim *et al.* [10], for example, have shown that the strength of metal matrix composites (6061/SiC) increased appreciably over that of the monolithic counterpart only when the volume fraction of the reinforcing particulates was increased over 28%. Friend [12] suggested that unless there is a critical volume fraction of the reinforcing phase in the matrix, the load transfer between the matrix and the reinforcement will not be effective and a concomitant strength improvement may not be realized. In view of these findings it becomes imperative to know, a priori, the minimum volume fraction of the reinforcement that may be required to realize the strength improvement. A theoretical model is therefore proposed in the present study. This model considers that with small volume fraction, the particulates act as imperfection in the matrix and cause high interfacial stress in the material. As a result, failure occurs at a stress level lower than a unreinforced metallic matrix can sustain.

Accordingly, the present study aimed to establish first the failure mode of the metallic matrix reinforced with hard ceramic particulates using inhomogeneity solution, and second to formulate the model incorporating the failure criterion established in the first part, to compute the minimum volume fraction that may be required to realize a strength improvement of the metallic matrix. Particular emphasis was placed in correlating the theoretical findings with the experimental results established previously.

## 2. The model

### 2.1. Failure mechanism and inhomogeneity solution

Particulate-reinforced metal matrix composites may exhibit different failure mechanisms, depending on the type of reinforcement used. For example, aluminium-based metal matrices can be reinforced by either soft particulates or hard particulates depending on the end

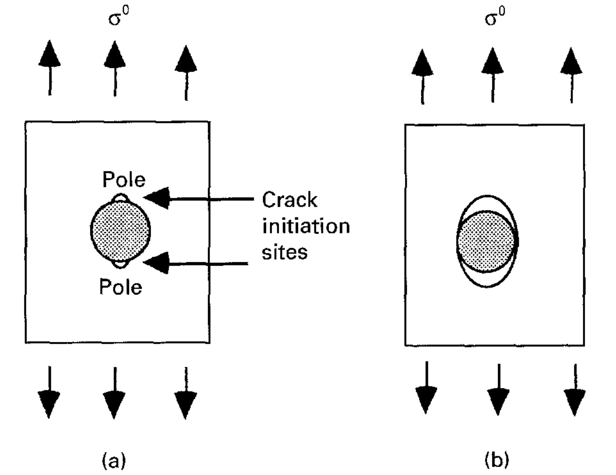


*Figure 1* Schematic illustration showing failure mechanism associated with hard particulates in a metal matrix composite: (a) crack initiation at two poles, and (b) crack propagation along the matrix/particulate interface.

use of the material [13, 14]. In related studies [15], it has been reported that a crack initiates at different sites with respect to the harder and softer particulate in the matrix under uniform far-field tensile loading. According to the aforementioned analysis, for soft inhomogeneity, the crack was shown to initiate at the equator due to the high stress concentration in that place. On the other hand, for a harder inhomogeneity, it was predicted that the crack will be initiated at two poles as a result of the high interfacial stresses (see Fig. 1a). Because, in the present study, an Al–Cu matrix reinforced by SiC particulates was investigated, the crack initiation mode associated with harder inhomogeneity was considered. Once initiated, the crack can possibly propagate in two directions: (a) into the ceramic reinforcement, (b) along the matrix/particulate interface. The propagation of the crack into the ceramic reinforcement is precluded, owing to the high strength of SiC particulates as well as the compressive stresses acting on the particulates under the given loading conditions. As a result, the crack will tend to propagate along the matrix/particulate interface (Fig. 1b). This suggestion is consistent with the studies of other investigators who observed that ~65% of the particulates in a spray-processed metal matrix composite debonded when the composite samples were subjected to 1% tensile plastic strain [3]. As the debonding continues, the load transfer from the metallic matrix to ceramic particulates will decrease rapidly, eventually leading to a catastrophic failure. Based on the above discussion, the interfacial stress, $\sigma^T_{33}(0, 0, \pm a)$, is identified to be responsible for the failure of MMC.

In order to find the maximum interfacial tensile stress, $\sigma^T_{33}(0, 0, \pm a)$, the solution of a spherical inhomogeneity embedded in an infinitely extended isotrophic material under uniform far-field tensile loading is considered following Eshelby's equivalent inclusion method [16]. In the present formulation, the terminology described elsewhere [17] will be used. An inclusion is defined as a sub-domain with the same modulus as that of the matrix and non-zero eigenstrain. On the other hand, an inhomogeneity is defined as a sub-domain with different elastic constants from that of the matrix. The equivalent inclusion method connects the inhomogeneity and inclusion by introducing proper eigenstrain. The elastic constants of the matrix, $C_{ijkl}$, and the inhomogeneity, $C^*_{ijkl}$, can be correlated with the eigenstrain, $\varepsilon^*_{ij}$, the strain disturbance, $\varepsilon_{ij}$, and the homogeneous strain in the absence of the inclusion, $\varepsilon^0_{ij}$, as

$$C^*_{ijkl}(\varepsilon^0_{kl} + \varepsilon_{kl}) = C_{ijkl}(\varepsilon^0_{kl} + \varepsilon_{kl} - \varepsilon^*_{kl}) \quad (1)$$

The total stress acting at any given point, $\sigma^T_{ij}$, is given by the following equation

$$\sigma^T_{ij} = \sigma^0_{ij} + \sigma_{ij} \quad (2)$$

where $\sigma^0_{ij}$ is the homogeneous stress in the absence of the inclusion and $\sigma_{ij}$ is the stress disturbance. These can further be expressed as

$$\sigma^0_{ij} = C_{ijkl}\varepsilon^0_{kl} \quad (3)$$

$$\sigma_{ij} = C_{ijkl}(\varepsilon_{kl} - \varepsilon^*_{kl}) \quad (4)$$

Equation 1 implies that the stress field inside the inhomogeneity can be replaced by the stress field produced by an inclusion assigned with a proper eigenstrain. In order to compute eigenstrain from Equation 1, the Eshelby inclusion solution is used. According to Eshelby's solution [16]

$$\varepsilon_{ij} = S_{ijkl}\varepsilon^*_{kl} \quad (5)$$

where $S_{ijkl}$ is the Eshelby tensor. For a spherical inclusion embedded in an isotropic medium, the Eshelby tensor is given as [17]

$$S_{1111} = S_{2222} = S_{3333} = \frac{7 - 5\nu}{15(1 - \nu)}$$

$$S_{1122} = S_{2233} = S_{3311} = \frac{5\nu - 1}{15(1 - \nu)}$$

$$S_{1212} = S_{2323} = S_{3131} = \frac{4 - 5\nu}{15(1 - \nu)} \quad (6)$$

where $\nu$ is Poisson's ratio.

With the eigenstrains, the stress distribution inside the inhomogeneity can be expressed using Equations 4 and 5 as

$$\sigma_{11} = -\mu\frac{16}{15(1 - \nu)}\varepsilon^*_{11} - 2\mu\frac{5\nu + 1}{15(1 - \nu)}\varepsilon^*_{22} - 2\mu\frac{5\nu + 1}{15(1 - \nu)}\varepsilon^*_{33}$$

$$\sigma_{12} = -2\mu\frac{7 - 5\nu}{15(1 - \nu)}\varepsilon^*_{12} \quad (7)$$

All other stress components can be obtained by the cyclic permutation of 1, 2 and 3. In the present approach, only uniform far field uniaxial loading in the $x_3$-direction ($\sigma^0$) is considered. The eigenstrain as a result of this loading can be expressed using equations 1, 3 and 5 as

$$\varepsilon_{11}^* = \varepsilon_{22}^* = \left(-\frac{5}{2\mu}A + \frac{1}{3K}B\right)\sigma^0$$

$$\varepsilon_{33}^* = \left(\frac{5}{\mu}A + \frac{1}{3K}B\right)\sigma^0 \qquad (8)$$

where $A$ and $B$ are expressed as

$$A = \frac{(\mu^* - \mu)(1 - \nu)}{(5\nu - 7)\mu - (8 - 10\nu)\mu^*}$$

$$B = \frac{(K^* - K)(1 - \nu)}{(4\nu - 2)K - (1 + \nu)K^*} \qquad (9)$$

Using the correlation of bulk modulus, $K$, and shear modulus, $\mu$, with Young's modulus, $E$, and Poisson's ratio, $\nu$

$$K = \frac{E}{3(1 - 2\nu)} \qquad (10a)$$

and

$$\mu = \frac{E}{2(1 + \nu)} \qquad (10b)$$

the total stress inside the inhomogeneity using Equations 2, 7 and 8 can be written as

$$\sigma_{11}^{\mathrm{T}} = \sigma_{22}^{\mathrm{T}} = (-F + G)\sigma^0$$

$$\sigma_{33}^{\mathrm{T}} = (2F + G)\sigma^0 \qquad (11)$$

where $F$ and $G$ are expressed as

$$F = \frac{1}{3}\left[1 + \frac{(5\nu - 7)(\mu^* - \mu)}{(5\nu - 7)\mu - (8 - 10\nu)\mu^*}\right]$$

$$G = \frac{1}{3}\left[1 + \frac{(4\nu - 2)(K^* - K)}{(4\nu - 2)K - (1 + \nu)K^*}\right] \qquad (12)$$

It may be noted that the stress inside the inhomogeneity is uniform. Therefore, the maximum interfacial tensile stress, $\sigma_{33}^{\mathrm{T}}(0, 0, \pm a)$, is equal to $\sigma_{33}^{\mathrm{T}}$ inside the inhomogeneity. Introducing a stress coefficient, $\kappa$, as

$$\kappa = 2F + G \qquad (13)$$

the expression for the interfacial stress shown in Equation 2 can be written as

$$\sigma_{33}^{\mathrm{T}} = \kappa\sigma^0 \qquad (14)$$

Using the material's constant values from Table I, the value of $\kappa$ was found to be 1.66. Equation 14 implies that the stress at the particulate/matrix interface will always be higher when compared to the far field loading for a metallic matrix reinforced with harder and stronger particulates.

Regarding the effect of volume fraction, it may be noted that for the same amount of far-field loading, the increase in volume fraction of the particulates will decrease the stress level that applies to each of the reinforcement particulates. This can be rationalized by analysing the average stress level in both the matrix and the particulates for a given volume fraction using the Mori–Tanaka theory [17]. According to the Mori–Tanaka theory, the average stresses in the matrix, $\langle\sigma_{ij}\rangle_{\mathrm{M}}$, and in the particulates, $\langle\sigma_{ij}\rangle_{\mathrm{P}}$, are related to the volume fraction as follows

$$\langle\sigma_{ij}\rangle_{\mathrm{M}} = -V_{\mathrm{f}}\sigma_{ij}^{\mathrm{s}} + \sigma_{ij}^0 \qquad (15)$$

$$\langle\sigma_{ij}\rangle_{\mathrm{P}} = (1 - V_{\mathrm{f}})\sigma_{ij}^{\mathrm{s}} + \sigma_{ij}^0 \qquad (16)$$

where $\sigma_{ij}^{s}$ is the stress calculated for a single particulate present in an infinite matrix under a uniform far-field loading, $\sigma_{ij}^0$. Equations 15 and 16 imply that with an increase in number of particulates (and hence the volume fraction) higher far-field loading would be required to cause crack initiation and propagation.

The formulation in the next section takes into consideration the findings of the inhomogeneity solution to compute the minimum volume fraction of the ceramic particulates required to realize a strength improvement of the MMC.

TABLE I Properties of reinforcement, matrix and composite material

| Materials | $E$(GPa) [18] | $\nu$ [18] | $\sigma_{\mathrm{uts}}$(MPa) [19] |
|---|---|---|---|
| Matrix (Al–Cu) | 73 | 0.33 | 410 |
| Reinforcement (SiC) | 450 | 0.17 | – |
| Composite ($V_{\mathrm{f}} = 11.1\%$) | – | – | 422 |

## 2.2. Formulation for critical volume fraction

In this section, a simple model is developed to explain the decrease in the ultimate tensile strength of the metal matrix composites reinforced with very small volume fraction of particulates and to predict the critical volume fraction of the particulates required in order to realize an improvement in strength of the MMCs when compared to that of the metallic matrix. The results of the model will simultaneously be analysed with the experimental observations. The formulation of the model is based on the following assumptions: (a) the reinforcing particulates are of spherical shape (aspect ratio = 1); (b) the reinforcing particulates act as load carriers until the onset of the debonding; and (c) the reinforcing particulates are uniformly distributed in the matrix.

The strength of MMCs may be estimated on the basis of the law of mixture as [12]

$$\sigma_{\mathrm{c}} = \sigma_{\mathrm{p}}V_{\mathrm{f}} + \sigma_{\mathrm{m}}^*(1 - V_{\mathrm{f}}) \qquad (17)$$

where $V_{\mathrm{f}}$ is the volume fraction of particulates, $\sigma_{\mathrm{c}}$ the composite strength, $\sigma_{\mathrm{p}}$ the strength of the reinforcing particulate, and $\sigma_{\mathrm{m}}^*$ the limiting composite strength ($V_{\mathrm{f}} \rightarrow 0$). It may be noted that

$$\sigma_{\mathrm{c}} = \begin{cases} \sigma_{\mathrm{mu}} & V_{\mathrm{f}} = 0 \\ \sigma_{\mathrm{m}}^* & V_{\mathrm{f}} \rightarrow 0 \end{cases} \qquad (18)$$

where $\sigma_{\mathrm{mu}}$ represents the ultimate tensile strength of the monolithic matrix material, while $\sigma_{\mathrm{m}}^*$ represents the ultimate tensile strength of the reinforced material with the volume fraction of ceramic reinforcement approaching zero.

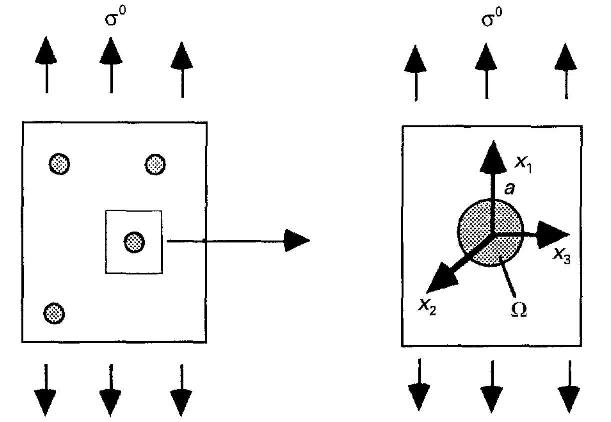


*Figure 2* Schematic diagram showing the inhomogeneity problem for a composite with $V_f \to 0$.

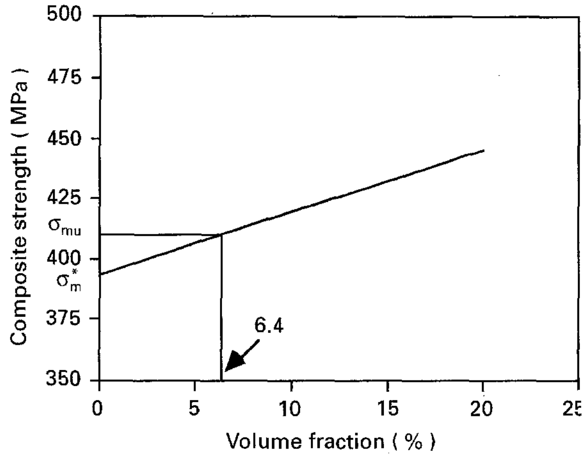


*Figure 3* Schematic diagram showing the variation in composite strength as a function of volume fraction of the reinforcement.

In view of the discussions made earlier regarding the interfacial crack initiation and propagation for a metallic matrix reinforced with harder particulates, and the supporting experimental observations made by various investigators [3, 13, 19], Equation 17 is modified to take into account the strength of the interface as the primary failure-governing mechanism. Hence, replacing $\sigma_p$ by $\sigma_i$

$$\sigma_c = \sigma_i V_f + \sigma^*_m(1 - V_f) \quad (19)$$

where $\sigma_i$ is the interfacial bond strength between the soft and ductile matrix and the hard and brittle reinforcement. It may be noted that when $\sigma_i$ is reached at the interface, the maximum stress in the particulate also reaches $\sigma_i$.

$\sigma^*_m$ can be determined by using, without losing generality, the configuration shown in Fig. 2. In other words, $\sigma^*_m$ is the failure tensile stress of an infinitely extended matrix material with an embedded spherical ceramic particulate. It may be noted that for $V_f \to 0$, each reinforcing particulate is located far away from each other and hence the interparticulate interactions and the stress disturbance at points in between particulates can be neglected. Considering that failure occurs at the onset of the interfacial debonding, the theoretical value of $\sigma^*_m$ and its relation to $\sigma_i$ can be determined using Equation 14 as

$$\sigma^T_{33} = \kappa\sigma^*_m = \sigma_i \quad (20)$$

Substituting the value of $\sigma^*_m$ from Equation 20 into Equation 19, we obtain

$$\sigma_c = \sigma_i V_f + \frac{\sigma_i}{\kappa}(1 - V_f) \quad (21)$$

Now using the volume fraction ($V_f = 11.1\%$) of the particulates in the MMC, the theoretically calculated value of $\kappa(\kappa = 1.66)$ and the composite strength shown in Table I, the interfacial strength, $\sigma_i$, was found to be 653 MPa. It may be noted that an independent estimate of $\sigma_i$ was not used in the present study because the interfacial behaviour differs strongly as a function of processing type, chemical composition of the matrix, secondary processing parameters and the heat treatment. Moreover, experimental determination cannot possibly be carried out as a result of discontinuous nature of the reinforcement and a relatively very small size of SiC particulates (3 μm) used in the present study.

Substituting the values of $\sigma_i$ and $\kappa$ into Equation 20, the ultimate tensile stress of the composite with $V_f \to 0$ is determined as $\sigma^*_m = 393$ MPa. It may be noted that this value is lower than the ultimate tensile strength of the matrix material $\sigma_m = 410$ MPa (see Table I). The results hence suggest that a critical volume fraction of the particulates is required for the strength of MMC to be equal to that of the matrix material (Fig. 3).

Now in the limit, when the volume fraction of reinforcement, $V_f$, equals the critical volume fraction, $V_{crit}$, the strength of the composite, $\sigma_c$, will be equal to that of the unreinforced matrix material, $\sigma_{mu}$. Hence we can rewrite Equation 21 as

$$\sigma_{mu} = \sigma_i V_{crit} + \frac{\sigma_i}{\kappa}(1 - V_{crit}) \quad (22)$$

which can again be rearranged as

$$V_{crit} = \frac{\kappa\sigma_{mu} - \sigma_i}{(\kappa - 1)\sigma_i} \quad (23)$$

Substituting the values of $\sigma_i$, $\sigma_{mu}$, and $\kappa$ in Equation 23 we obtain a critical volume fraction $V_{crit} = 6.4\%$ for the SiC particulate-reinforced Al–Cu matrix composite. The results of the above calculation are listed in Table II.

It may be noted that a small difference (4.7%) between the calculated (6.4%) and the actual volume fraction (11.1%) is consistent with the marginal improvement in the strength that was experimentally observed. The computed value of the critical volume fraction, however, should be considered as a lower-bound estimate, because, in practice, a completely uniform distribution of reinforcing particulates in the metallic matrix is difficult to achieve, and hence there are always clusters or agglomeration sites present in the matrix which promote early crack nucleation. The calculated value of the critical volume fraction, however, provides an insight into the minimum volume fraction of reinforcement that is required to realize an improvement in the strength in discontinuously

TABLE II Input parameters and results of the model

| Variable | Unit | Value |
|---|---|---|
| $\sigma_i$ | MPa | 653 |
| $\sigma_{mu}$ | MPa | 410 |
| $\sigma_m^*$ | MPa | 393 |
| $V_{crit}$ | % | 6.4 |

reinforced MMCs. Moreover, inspection of Equation 23 reveals some interesting trends. First Equation 23 suggest that the higher the interfacial strength the lower is the volume fraction of the ceramic reinforcement that may be required to realize a strength improvement in discontinuously reinforced MMCs. This observation is true, because a high value of $\sigma_i$ is indicative of a more effective matrix–reinforcement load transfer. Hence, the MMC will need a smaller volume fraction of reinforcement relative to one with a poorly bonded reinforcement (i.e. low $\sigma_i$). Secondly, Equation 23 suggests that an increase in the matrix strength, $\sigma_{mu}$, is accompanied by an increase in critical volume fraction. This is consistent with the results reported by McDanels [20] for 20 vol % $SiC_W$/Al in which he noted that the strength improvement that was realized in a 6061 MMC was higher than that noted for 2124 and 7075 MMCs, consistent with the lower matrix strength of the 6061 alloy relative to alloys 2124 and 7075.

## 3. Conclusions

1. For a ductile metallic matrix reinforced with hard particulates subjected to far-field uniaxial loading, the matrix/particulate interface failure mechanism will dominate as a result of high interfacial stress at the two poles.

2. The stress at the matrix/particulate interface may exceed the interfacial debonding stress for $V_f \rightarrow 0$ even when the far-field loading stress is lower than the strength of the metallic matrix.

3. Preliminary results obtained for the strengthening behaviour, on the basis of a simple numerical formulation, suggest that a minimum of 6.4 vol % SiC particulates is required in order to realize a strength improvement for the Al–Cu matrix material investigated in this study.

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